

**JEE MAIN 2025**  
**Sample Paper - 8**

**Time Allowed: 3 hours**

**Maximum Marks: 300**

**General Instructions:**

1. There are three subjects in the question paper consisting of Physics (Q. no. 1 to 25), Chemistry (Q. no. 26 to 50), and Mathematics (Q. no. 51 to 75).
2. Each subject is divided into two sections. Section A consists of 20 multiple-choice questions & Section B consists of 5 numerical value-type questions.
3. There will be only one correct choice in the given four choices in Section A. For each question for Section A, 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice questions and zero marks will be awarded for not attempted questions.
4. For Section B questions, 4 marks will be awarded for correct answers and zero for unattempted and incorrect answers.
5. Any textual, printed, or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
6. All calculations/written work should be done in the rough sheet is provided with the Question Paper.

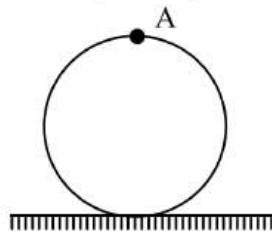


**SECTION – I**  
**(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

**Marking scheme:** +4 for correct answer, 0 if not attempted and -1 if not correct.

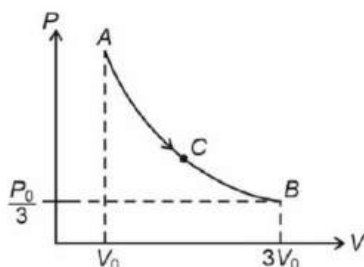
1. In a new system of units, if energy (E), velocity (V) and time (T) are chosen as fundamental quantities, then dimensional formula of force is  $[E^\alpha V^\beta T^\gamma]$ . The value of  $\alpha + \beta - \gamma$  is  
 1) 2                      2) -2                      3) 0                      4) 1
2. A bar magnet is held perpendicular to a uniform magnetic field. If the couple acting on the magnet is to be halved by rotating it, then the angle by which it is to be rotated is  
 1)  $30^\circ$                       2)  $45^\circ$                       3)  $60^\circ$                       4)  $90^\circ$
3. A block is moving on horizontal rough surface. It has velocity  $v_0 \hat{i}$  at  $x = 0.1$  m. The coefficient of friction varies with distance (x) from origin as  $\mu = \frac{1}{2x^2}$ . The minimum value of  $v_0$  so that it never stops is  
 1) 5 m/s                      2) 7 m/s                      3) 8.5 m/s                      4) 10 m/s
4. A disc (m, R) is doing pure rolling motion on a horizontal surface. Radius of curvature of trajectory followed by point A at topmost point is



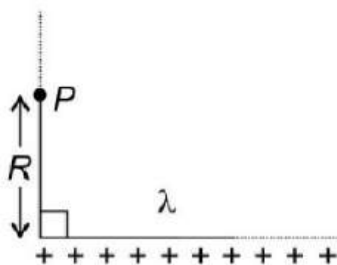
- 1) R                      2) 2R                      3) 4R                      4)  $2\sqrt{2}R$
5. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to  $T_M$ . If the Young's modulus of the material of the wire is Y, then  $\frac{1}{Y}$  is equal to (g = gravitational acceleration)

- 1)  $\left[ \left( \frac{T_M}{T} \right)^2 - 1 \right] \frac{Mg}{A}$     2)  $\left[ 1 - \left( \frac{T_M}{T} \right)^2 \right] \frac{A}{Mg}$     3)  $\left[ 1 - \left( \frac{T}{T_M} \right)^2 \right] \frac{A}{Mg}$     4)  $\left[ \left( \frac{T_M}{T} \right)^2 - 1 \right] \frac{A}{Mg}$

6. An ideal monatomic gas undergoes isothermal expansion from state  $A$  to  $B$ . Work done by the gas from  $A \rightarrow B$  is double of work done by the gas from  $A \rightarrow C$ . Pressure at point  $C$  is

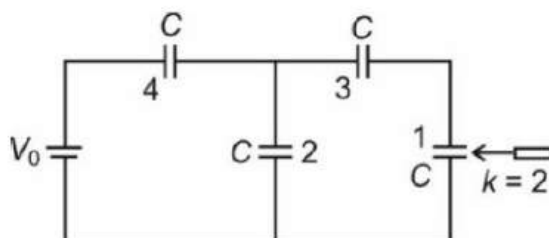


- 1)  $\frac{P_0}{\sqrt{3}}$       2)  $\frac{P_0}{3}$       3)  $\frac{2P_0}{3}$       4)  $\frac{2P_0}{\sqrt{3}}$
7. One mole of an ideal gas undergoes a process given by,  $T = T_0 + \alpha V^2$  where  $T$  is the temperature and  $V$  is the volume of gas. Volume of gas when its pressure is least is [Where  $T_0$  and  $\alpha$  are positive constants].
- 1)  $\sqrt{T_0 / 2\alpha}$       2)  $\sqrt{T_0 / \alpha}$       3)  $\sqrt{\frac{2T_0}{\alpha}}$       4)  $2\sqrt{\frac{T_0}{\alpha}}$
8. Two particle A and B are performing SHM with amplitude  $A_0$ , and time period  $T$  about the same mean position. At  $t = 0$ , A is at mean position and B is at distance  $\frac{A_0}{2}$  from mean position and is going towards mean position. At what time they will be at maximum separation? (At  $t = 0$ , direction of velocities of A and B are same)
- 1)  $\frac{T}{12}$       2)  $\frac{T}{8}$       3)  $\frac{T}{30}$       4)  $\frac{T}{24}$
9. Electric field at point 'P' due to long rod having uniform charge density,  $\lambda$  as shown is

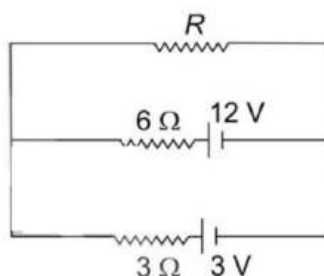


- 1)  $\frac{\lambda}{4\pi\epsilon_0 R}$       2)  $\frac{\lambda}{2\sqrt{2}\pi\epsilon_0 R}$       3)  $\frac{\lambda}{2\pi\epsilon_0 R}$       4)  $\frac{\lambda}{\sqrt{2}\pi\epsilon_0 R}$

10. For the circuit shown in figure, dielectric slab of dielectric constant  $k = 2$  is inserted in space between the plates of capacitor 1. As the dielectric is inserted

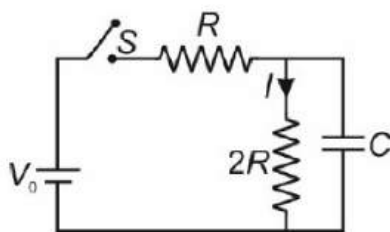


- 1) Potential across capacitor 1 increases
  - 2) Charge on capacitor 3 decreases
  - 3) Charge on capacitor 4 decreases
  - 4) Charge on capacitor 2 decreases
11. For the circuit shown in figure, value of resistance  $R$  is adjusted so that power delivered to resistor,  $R$  is maximum and is equal to  $P_0$ . Value of  $P_0$  is

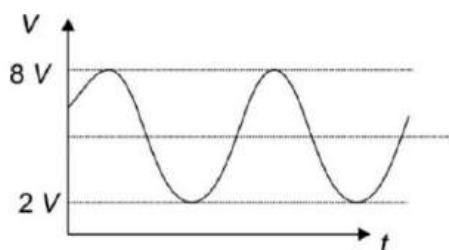


- 1) 6.0W                      2) 3.0W                      3) 9.0W    4) 4.5W
12. STATEMENT-1 : Inductance plays same role in the electrical circuits as mass plays in the mechanical circuits.  
STATEMENT-2 : Greater the value of inductance, harder it is to change the current in the circuit.
- (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
  - (2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
  - (3) Statement-1 is True, Statement-2 is False
  - (4) Statement- 1 is False, Statement- 2 is True

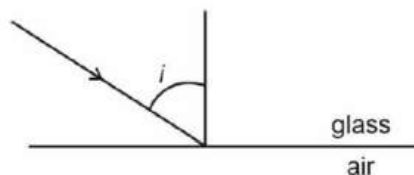
13. For the RC circuit shown in figure, switch 'S' is closed at time  $t = 0$ . Current  $I$  at  $t = \frac{2RC}{3} \ln 3$ , is



- 1)  $\frac{V_0}{9R}$       2)  $\frac{2V_0}{9R}$       3)  $\frac{2V_0}{R}$       4)  $\frac{V_0}{4R}$
14. A sinusoidally varying source voltage is given as a function of time as shown. RMS value of voltage is



- 1)  $\sqrt{\frac{59}{2}}V$       2)  $\frac{7}{\sqrt{2}}V$       3) 4V      4) 6V
15. Magnetic field associated with electromagnetic wave whose electric field is given by  $\vec{E} = 2.1 \sin(3 \times 10^8 t - 1.8Z) \hat{i} \text{ N/C}$ , is
- 1)  $\vec{B} = 1.26 \times 10^{-8} \sin(3 \times 10^8 t - 1.8Z) \hat{j} \text{ T}$       2)  $\vec{B} = 1.8 \times 10^{-8} \sin(3 \times 10^8 t - 1.8Z) \hat{j} \text{ T}$
- 3)  $\vec{B} = 0.7 \times 10^{-8} \sin(3 \times 10^8 t - 1.8Z) \hat{j} \text{ T}$       4)  $\vec{B} = -1.26 \times 10^{-8} \sin(3 \times 10^8 t - 1.8Z) \hat{j} \text{ T}$
16. A ray of light travelling from glass to air is incident at angle  $i$ . Maximum angle of deviation suffered for any angle of incidence is  $\frac{\pi}{2}$ , then refractive index of glass is



- 1)  $\sqrt{2}$       2)  $\sqrt{3}$       3) 2      4)  $4/3$

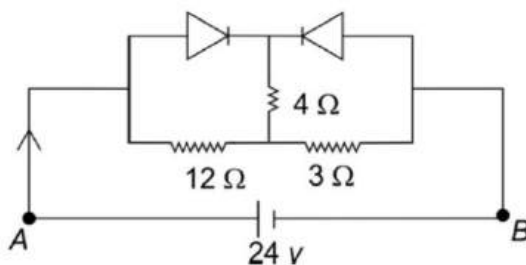
17. In a standard YDSE, the distance between the slits varies with time  
 $(t) \text{ as } (d) = d_0 + a_0 \sin(\omega t)$ . The difference between the largest fringe width and the smallest fringe width over time is given as ( $\lambda$  is wavelength of monochromatic light used,  $D$  is the screen distance from slits.  $D \gg d$ )

1)  $\frac{2\lambda D a_0}{d_0^2 - a_0^2}$       2)  $\frac{2\lambda D d_0}{d_0^2 - a_0^2}$       3)  $\frac{\lambda D}{d_0 + a_0}$       4)  $\frac{\lambda D a_0}{d_0^2 - a_0^2}$

18. In a hydrogen atom, electron jumps from 4th excited state to 2<sup>nd</sup> excited state. Wavelength of photon emitted is [ $R$  : Rydberg constant]

1)  $\frac{225}{16R}$       2)  $\frac{225}{4R}$       3)  $\frac{100}{21R}$       4)  $\frac{100}{4R}$

19. In the circuit with ideal diodes as shown, current (in A) through battery is



1) 3      2) 5      3) 6      4) 4

20. In a Vernier calipers, one main scale division is 1mm and 9 main scale divisions are equal to 10 vernier scale divisions. When nothing is put between jaws of the calipers, zero of the Vernier scale lies to the right side of zero of the main scale and the 2<sup>nd</sup> division of the Vernier scale coincides with a main scale division. While measuring inner diameter of a hollow cylinder the zero of Vernier scale lies between 1.7cm and 1.8cm of the main scale. Also, 8<sup>th</sup> division of Vernier scale coincides with a main scale division, inner diameter of the cylinder is

1) 17.2mm      2) 17.4mm      3) 17.6mm      4) 17.8mm

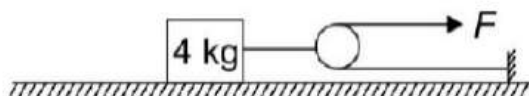
## SECTION-II (NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals, **Mark nearest Integer only.**

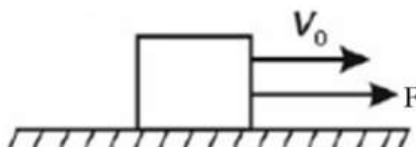
**Marking scheme: +4 for correct answer, -1 in all other cases.**

21. Pitch of a screw gauge is 1 mm and its cap is divided into 100 divisions. When nothing is placed between studs of the screw gauge, zero of circular scale is 8 divisions above the reference line and zero of the main scale is not visible. Now, when a cylindrical wire is placed between its studs the main scale reading is 2 divisions and 15<sup>th</sup> division of circular scale coincides with reference line. Diameter of the wire is  $x$  mm. Find the value of  $100x$ .

22. If coefficient of friction between all the surfaces is 0.50, then force,  $F$  (in N) required to move the block of mass 4 kg is [ $g = 10 \text{ m/s}^2$ ]



23. A block of mass ' $m$ ' placed on smooth horizontal surface is acted upon by a horizontal force as shown, delivering constant power ' $P$ '. If velocity of block changes from  $v_0$  to  $2v_0$ , then time taken is  $\frac{qmv_0^2}{nP}$ , find the value of  $q \times n$  is



24. Mass density of a disc is given by  $\sigma = \sigma_0 \frac{r}{R}$ , where  $\sigma_0$  is constant,  $r$  is distance from centre and  $R$  is radius of disc. Moment of inertia of disc about an axis passing through centre and perpendicular to plane of disc is  $\frac{n\pi\sigma_0 R^4}{m}$ . Find the value of  $n \times m$  is
25. The minimum and maximum distances of a planet from sun, revolving around the sun, are  $x_0$  and  $2x_0$ . If the maximum speed is  $v_0$ , then minimum acceleration of planet during motion is  $\frac{nv_0^2}{mx_0}$ . Find  $n \times m$

**SECTION – I**  
**(SINGLE CORRECT ANSWER TYPE)**

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.**

26. Titanium oxide ( $\text{TiO}_2$ ) is heated in stream of hydrogen to give water and new oxide  $\text{Ti}_x\text{O}_y$ . If 1.6gm  $\text{TiO}_2$  produces 1.44 gm  $\text{Ti}_x\text{O}_y$ , (atomic mass Ti = 48, O = 16), The sum of 'x' & 'y' is
- 1) 3                      2) 5                      3) 7                      4) 8
27. The standard reduction potential for  $\text{Zn}^{+2}/\text{Zn}$ ;  $\text{Ni}^{+2}/\text{Ni}$  and  $\text{Fe}^{+2}/\text{Fe}$  are  $-0.76\text{V}$ ,  $-0.23\text{V}$ ,  $-0.44\text{V}$  respectively. The reaction  $\text{X} + \text{Y}^{+2} \rightarrow \text{X}^{+2} + \text{Y}$  will be non-spontaneous when :
- |       |    |    |
|-------|----|----|
|       | X  | Y  |
| (I)   | Ni | Fe |
| (II)  | Ni | Zn |
| (III) | Fe | Zn |
| (IV)  | Zn | Ni |
- 1) I, II, IV              2) I, II, III              3) II, III, IV              4) I, III, IV
28. Consider the following chemical reaction and the corresponding kinetic data showing the initial reaction rate as a function of the initial concentrations of the reactants:
- $$\text{H}_3\text{AsO}_4(\text{aq}) + 2\text{H}_3\text{O}^+(\text{aq}) + 3\text{I}^-(\text{aq}) \rightarrow \text{HAsO}_2(\text{aq}) + \text{I}_3^-(\text{aq}) + 4\text{H}_2\text{O}(\text{liq})$$
- | Initial Rate $\times 10^{-5}$<br>(M/sec) | $[\text{H}_3\text{AsO}_4]$ | $[\text{H}_3\text{O}^+]$ | $[\text{I}^-]$ |
|------------------------------------------|----------------------------|--------------------------|----------------|
| 3.7                                      | 0.001                      | 0.01                     | 0.10           |
| 7.4                                      | 0.001                      | 0.01                     | 0.20           |
| 7.4                                      | 0.002                      | 0.01                     | 0.10           |
| 3.7                                      | 0.002                      | 0.005                    | 0.20           |
- Using the data, establish the correct reaction composite order.
- 1) 1                      2) 2                      3) 3                      4) 4

29. A 0.4m aqueous solution of  $\text{Na}_x\text{A}$  has freezing point  $-3.72^\circ\text{C}$ . If  $K_f(\text{H}_2\text{O})$  is  $1.86\text{K kg mol}^{-1}$ . The value of 'X' is (salt is 100% ionized).

- 1) 2                      2) 3                      3) 4                      4) 6

30. Which of the following below electronic configuration of lanthanides is related to the formation of stable +2 oxidation state.

- 1)  $[\text{Xe}]4f^7, 5d^1, 6s^2$     2)  $[\text{Xe}]4f^{14}, 5d^1, 6s^2$     3)  $[\text{Xe}]4f^7, 6s^2$             4)  $[\text{Xe}]4f^1, 5d^1, 6s^2$

31.  $[\text{Co}(\text{NH}_3)_4(\text{NO}_2)_2]\text{Cl}$  exhibits:

- 1) linkage isomerism, geometrical isomerism and optical isomerism  
2) linkage isomerism, ionization isomerism and optical isomerism  
3) linkage isomerism, ionization isomerism and geometrical isomerism  
4) ionization isomerism, geometrical isomerism and optical isomerism

32. The enthalpy of combustion of propane ( $\text{C}_3\text{H}_8$ ) gas in terms of given data is :  
Bond energy (kJ/mol)

|                        |                        |                        |                        |                        |
|------------------------|------------------------|------------------------|------------------------|------------------------|
| $\epsilon \text{ C—H}$ | $\epsilon \text{ O=O}$ | $\epsilon \text{ C=O}$ | $\epsilon \text{ O—H}$ | $\epsilon \text{ C—C}$ |
| + $x_1$                | + $x_2$                | + $x_3$                | + $x_4$                | + $x_5$                |

Resonance energy of  $\text{CO}_2$  is  $-z$  kJ/mol and  $\Delta H_{\text{vaporization}} [\text{H}_2\text{O}(l)]$  is  $y$  kJ/mol.

- 1)  $8x_1 + 2x_5 + 5x_2 - 6x_3 - 8x_4 - 4y - 3z$   
2)  $6x_1 + x_5 + 5x_2 - 3x_3 - 4x_4 - 4y - 3z$   
3)  $8x_1 + 2x_5 + 5x_2 - 6x_3 - 8x_4 - y - z$   
4)  $8x_1 + x_5 + 5x_2 - 6x_3 - 8x_4 - 4y + 3z$

33.  $\frac{N_0}{2}$  atoms of X (g) are converted into  $\text{X}^+$  (g) by absorbing  $E_1$  energy.  $2N_0$  atoms of X (g) are converted into  $\text{X}^-$  (g) by releasing  $E_2$  energy. Calculate ionisation enthalpy and electron gain enthalpy of X(g) per atom.

- 1)  $\text{I.E.} = \frac{2E_1}{N_0}$ ,  $\Delta_{\text{eg}}\text{H} = -\frac{E_2}{2N_0}$                       2)  $\text{I.E.} = -\frac{E_2}{2N_0}$ ,  $\Delta_{\text{eg}}\text{H} = \frac{2E_1}{N_0}$   
3)  $\text{I.E.} = \frac{E_1}{2N_0}$ ,  $\Delta_{\text{eg}}\text{H} = -\frac{E_2}{2N_0}$                       4)  $\text{I.E.} = \frac{N_0}{2E_1}$ ,  $\Delta_{\text{eg}}\text{H} = -\frac{2N_0}{E_2}$



34. For an octahedral complex, which of the following d-electron configuration will give maximum CFSE?

- 1) high spin,  $d^6$       2) low spin,  $d^5$       3) low spin,  $d^4$       4) high spin,  $d^7$

35. In  $ICl_2^+$ ,  $ICl_2^-$  and  $ICl_4^-$  sum of the bond pairs and lone pairs on each iodine atom in the given ionic species are

- 1) 2,2 and 4      2) 2,3 and 2      3) 4,5 and 4      4) 4,5 and 6

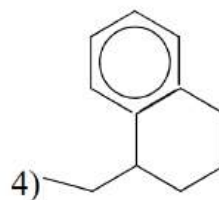
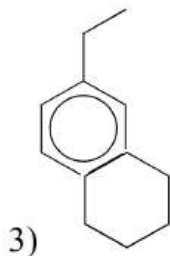
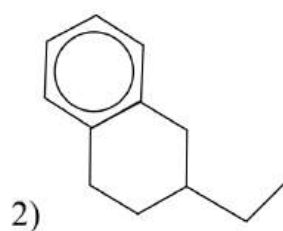
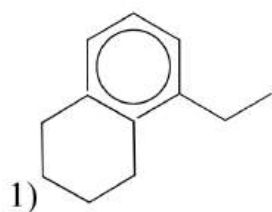
36. 0.2 g of an organic compound was analysed by kjeldahl's method. Ammonia evolved was absorbed in 60mL  $N/5 H_2SO_4$ . Unused acid required 40 mL of  $N/10 NaOH$  for complete neutralisation. Find the percentage of nitrogen in the compound.

- 1) 70 %      2) 56 %      3) 46 %      4) 66 %

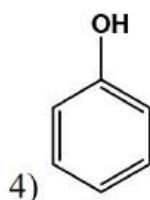
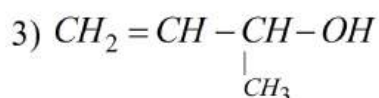
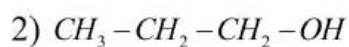
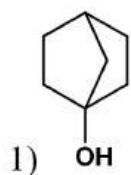
37. The correct order of increasing  $C-O$  bond length of  $CO$ ,  $CO_3^{2-}$ ,  $CO_2$  is

- 1)  $CO_3^{2-} < CO_2 < CO$       2)  $CO_2 < CO_3^{2-} < CO$   
3)  $CO < CO_3^{2-} < CO_2$       4)  $CO < CO_2 < CO_3^{2-}$

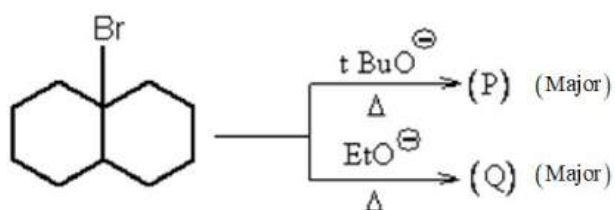
38. The major product in the following reaction.



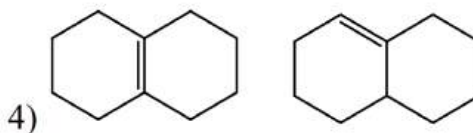
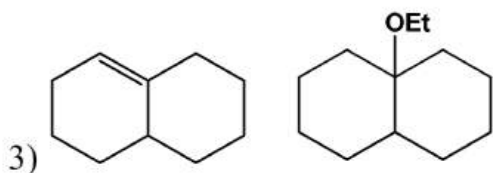
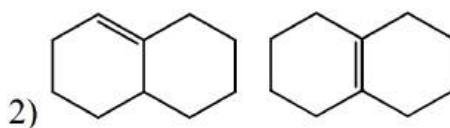
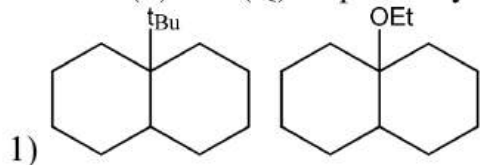
39. Which of the following can give immediate turbidity on treatment with Lucas Reagent?



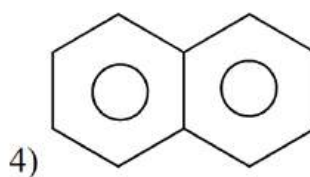
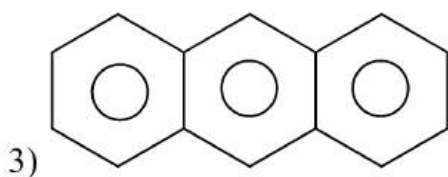
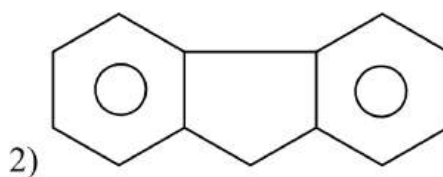
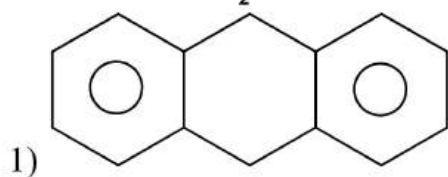
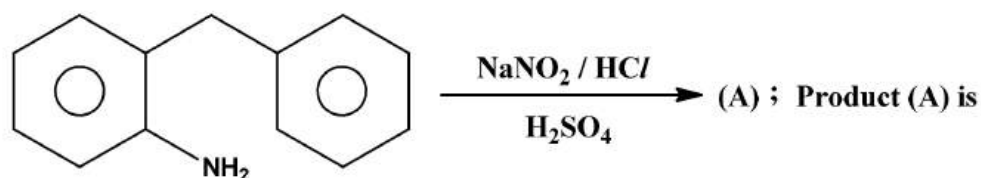
40.



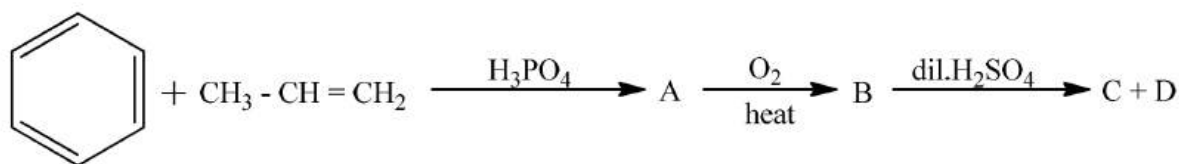
Product (P) and (Q) respectively



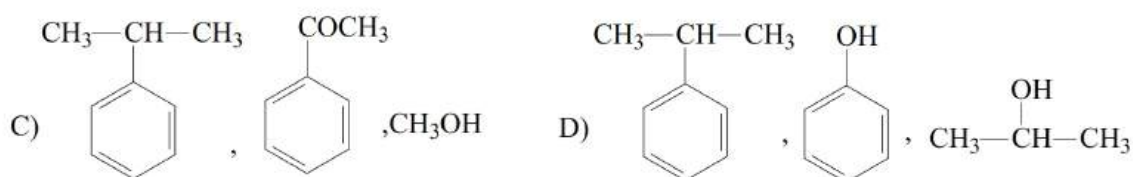
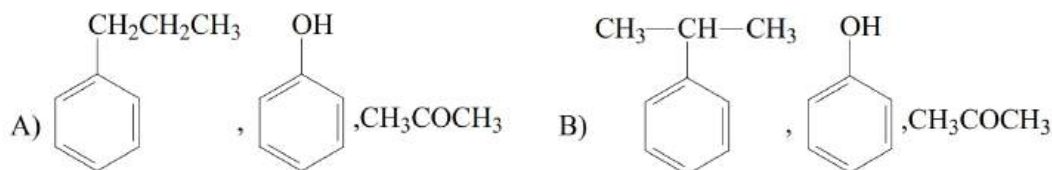
41.



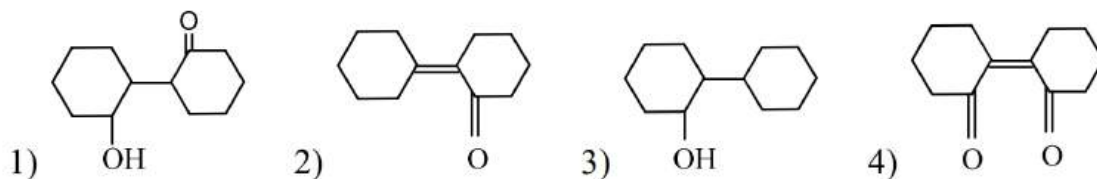
42.



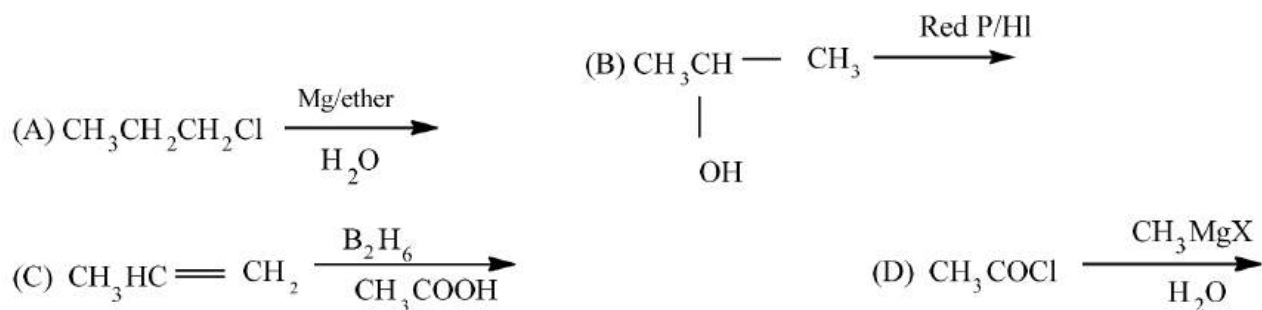
A, C, D are



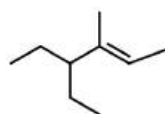
43. Which is the product formed when cyclohexanone undergoes aldol condensation followed by heating?



44. Which of the following reactions will not give propane?



45. The IUPAC name of the following compound is:



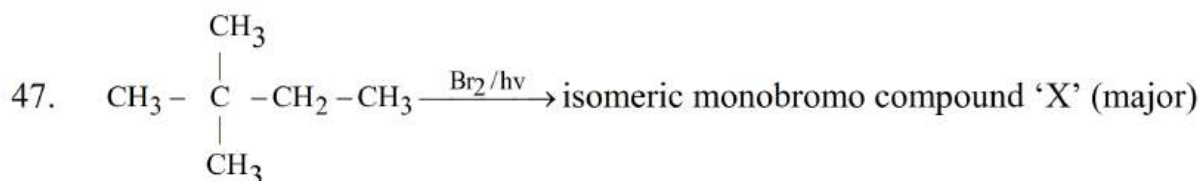
- 1) 3-ethyl-4-methylhex-4-ene      2) 4,4-diethyl-3-methylbut-2-ene  
3) 4-methyl-3-ethylhex-4-ene      4) 4-ethyl-3-methylhex-2-ene

**SECTION-II**  
**(NUMERICAL VALUE ANSWER TYPE)**

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals, **Mark nearest Integer only.**

**Marking scheme: +4 for correct answer, -1 in all other cases.**

46. The angular velocity of an electron occupying the second Bohr orbit of  $\text{He}^+$  ion is  $2.067 \times 10^x$  rad/sec, what is x \_\_\_\_\_



The position of the bromine atom in the major product 'X' is \_\_\_\_ (as per IUPAC Nomenclature)

48. Number of dipeptides possible using alanine, glycine and tyrosine is 'X' then what is the value of  $5x$ ?
49. Sum of basicity of  $\text{H}_3\text{PO}_4$ ,  $\text{H}_3\text{PO}_3$  and  $\text{H}_3\text{PO}_2$  is equal to
50. A buffer solution is formed by mixing 100 mL 0.01 M  $\text{CH}_3\text{COOH}$  with 200 mL 0.02 M  $\text{CH}_3\text{COONa}$ . If this buffer solution is made to 1.0 L by adding 700 mL of water, pH will change by a factor of 'x' then what is the value of  $x+12$ ?



SECTION – I

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which **ONLY ONE** option can be correct.

**Marking scheme: +4 for correct answer, 0 if not attempted and -1 if not correct.**

51. Statement-1: The point  $Z$  lies on the circle  $|Z - 2 - i| = 1$ . The point  $Z_0$  on the circle with maximum argument is given by  $Z_0 = 2(\cos \theta + i \sin \theta)$  where  $\tan \theta = \frac{4}{5}$  and

Statement-2: Point  $Z$  on the circle  $|Z - 2 - i| = 1$  nearest to the origin has modulus  $(\sqrt{5} - 1)$

1) Statement 1 is true, statement 2 is true, statement 2 is a correct explanation of statement 1

2) Statement 1 is true, statement 2 is true, statement 2 is not correct explanation of statement 1

3) Statement 1 is true, statement 2 is false

4) Statement 1 is false, statement 2 is true.

52. If  $a = \cot 80^\circ$ ,  $b = \cot 60^\circ$  and  $c = \cot 40^\circ$ , then the value of  $\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix}$  is equal to

1) 2

2) 1

3) 4

4) 3

53.  $f(x)$  is a cubic polynomial  $x^3 + ax^2 + bx + c$  such that  $f(x) = 0$  has three distinct integral roots and  $f(g(x)) = 0$  does not have real roots, where  $g(x) = x^2 + 2x - 5$  then the minimum value of  $a + b + c$  is

1) 504

2) 532

3) 719

4) 764

54. In how many ways can 17 persons depart from railway station in 2 cars and 3 autos given that 2 particular persons depart by the same car (4 persons can sit in a car and 3 persons can sit in an auto) is

1)  $\frac{15!}{2!4!(3!)^3}$

2)  $\frac{16!}{(2!)^2 4!(3!)^3}$

3)  $\frac{17!}{2!4!(3!)^3}$

4)  $\frac{15!}{4!(3!)^3}$

55. In a test an examinee either guess or copies or knows the answer to a multiple choice question with four choices of which only one option is correct. The probability that he makes a guess is  $\frac{1}{3}$  and the probability that he copies the answer is  $\frac{1}{6}$ , the probability that his answer is correct given that he copied is  $\frac{1}{8}$ , then the probability that he know the answer to the question given that he correctly answered it is

- 1)  $\frac{24}{29}$                       2)  $\frac{23}{29}$                       3)  $\frac{22}{29}$                       4)  $\frac{21}{29}$

56. A random variable  $X$  has the probability distribution

|         |      |      |      |      |      |      |      |      |
|---------|------|------|------|------|------|------|------|------|
| $X:$    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    |
| $P(X):$ | 0.15 | 0.23 | 0.12 | 0.10 | 0.20 | 0.08 | 0.07 | 0.05 |

For the events  $E = \{X \text{ is a prime number}\}$  and  $F = \{X < 4\}$  then probability of  $P(E \cup F)$

- 1) 0.87                      2) 0.77                      3) 0.35                      4) 0.50

57. If  $g(x) = \max |y^2 - xy|$ ,  $0 \leq y \leq 1$  then minimum value of  $g(x)$  for real  $x$  is  $a - 2\sqrt{b}$  ( $a, b$  are natural numbers) then  $a + b =$  \_\_\_\_\_

- 1) 5                      2) 11                      3) 24                      4) 2

58. A lamp of negligible height is placed on the ground  $m_1$  mt away from a wall. A man  $m_2$  mt tall is walking at a speed of  $\frac{m_1}{10}$  mt / sec from the lamp to the nearest point on the wall.

When he is midway between the lamp and the wall, the rate of change in the length of his shadow on the wall is

- 1)  $\frac{-5m_2}{2} m/s$                       2)  $\frac{-2m_2}{5} m/s$                       3)  $\frac{-m_2}{2} m/s$                       4)  $\frac{-m_2}{5} m/s$

59. If  $f(x) = \prod_{k=1}^{\infty} \left( \frac{1 + 2 \cos \left( \frac{2x}{3^k} \right)}{3} \right)$ , then number of points where

$[xf(x)] + |xf(x)| + (x-1)|x^2 - 3x + 2|$  is non-differentiable in  $x \in (0, 3\pi)$  is equal to

(where  $[.]$  denotes greatest integer function)

- 1) 5                      2) 6                      3) 4                      4) 8

60. A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s. The total number of ways in which the person can go up the stairs is
- 1) 75                      2) 79                      3) 85                      4) 89
61. Let  $\vec{k}, \vec{l}, \vec{m}, \vec{n}$  are four distinct unit vectors in a three-dimensional space such that  $\vec{k} \cdot \vec{l} = \vec{l} \cdot \vec{m} = \vec{m} \cdot \vec{k} = \vec{n} \cdot \vec{l} = \vec{n} \cdot \vec{m} = \frac{-1}{11}$ . If the value of  $\vec{k} \cdot \vec{n}$  can be expressed as  $\frac{-A}{B}$ , where A, B are coprime positive integers, then the unit digit of the value of (A+B) is \_\_\_\_\_
- 1) 0                      2) 4                      3) 6                      4) 8
62. The value of  $\int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$  is
- 1)  $\frac{1}{2\sqrt{11}} \ln \frac{(\sqrt{11}+1)^2}{10}$     2)  $\frac{1}{\sqrt{11}} \ln \frac{(\sqrt{11}+1)^2}{10}$     3)  $\frac{1}{2\sqrt{11}} \ln \frac{(\sqrt{11}-1)^2}{10}$     4)  $\frac{1}{\sqrt{11}} \ln \frac{(\sqrt{11}-1)^2}{10}$
63. The area bounded by the two branches of curve  $(y-x)^2 = x^3$  and the straight line  $x=1$  is
- 1) 1/5 sq.unit            2) 3/5 sq.unit            3) 4/5 sq.unit            4) 8/5 sq.unit
64. Statement-1: Period of  $f(x) = \sin 3x \cos [3x] - \cos 3x \sin [3x]$  where  $[ ]$  denotes the greatest integer function, is  $\frac{2\pi}{3}$
- Statement-2: Period of  $\{x\}$  where  $\{ \}$  denotes the fractional part of  $x$ , is 1
- 1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- 2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- 3) Statement-1 is True, Statement-2 is False
- 4) Statement-1 is False, Statement-2 is True

65. Let  $a_1, a_2, a_3, \dots, a_{101}$  are in GP with  $a_{51} = 25$  and  $\sum_{i=1}^{101} a_i = 125$ , then the value of  $\sum_{i=1}^{101} \left(\frac{1}{a_i}\right)$  equals to
- 1) 5                      2) 1/5                      3) 1/25                      4) 1/125
66. The last term in the binomial expansion of  $\left(\sqrt[3]{2} - \frac{1}{\sqrt{2}}\right)^n$  is  $\left(\frac{1}{3\sqrt[3]{9}}\right)^{\log_3 8}$  then the 5<sup>th</sup> term from the beginning is
- 1)  $^{10}C_6$                       2)  $2 \cdot ^{10}C_4$                       3)  $\frac{1}{2} \cdot ^{10}C_4$                       4)  $\frac{1}{3} \cdot ^{10}C_4$
67. The mean and variance of 7 observations are 8 and 16 respectively. If five observations out of them are 2, 4, 10, 12, 14 then the product of the two observations is
- 1) 48                      2) 49                      3) 36                      4) 24
68. If  $(a-b)\sin(\theta+\phi) = (a+b)\sin(\theta-\phi)$  and  $a \tan \frac{\theta}{2} - b \tan \frac{\phi}{2} = c$  then the value of  $\sin \phi$  is equal to
- 1)  $\frac{2ab}{a^2 - b^2 - c^2}$                       2)  $\frac{2bc}{a^2 - b^2 - c^2}$                       3)  $\frac{2bc}{a^2 - b^2 + c^2}$                       4)  $\frac{2ab}{a^2 - b^2 + c^2}$
69. The value of  $\lim_{x \rightarrow 0^+} \frac{-1 + \sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + \dots}}}}{-1 + \sqrt{x^3 + \sqrt{x^3 + \sqrt{x^3 + \dots}}}}$
- 1) 1                      2) 1/2                      3) 1/4                      4) -1
70.  $\sin^{-1}\left(\frac{x^2}{4} + \frac{y^2}{9}\right) + \cos^{-1}\left(\frac{x}{2\sqrt{2}} + \frac{y}{3\sqrt{2}} - 2\right)$  is equal to
- 1)  $\frac{\pi}{2}$                       2)  $\pi$                       3)  $\frac{\pi}{\sqrt{2}}$                       4)  $\frac{3\pi}{2}$

## SECTION-II (NUMERICAL VALUE ANSWER TYPE)

This section contains 5 questions. The answer to each question is a Numerical value. If the Answer in the decimals, **Mark nearest Integer only.**

**Marking scheme: +4 for correct answer, -1 in all other cases.**

71. Let  $z_1 = 10 + 6i$  and  $z_2 = 4 + 6i$ . If  $z$  is any complex number such that the argument of  $(z - z_1)/(z - z_2)$  is  $\frac{\pi}{4}$  and if  $|z - 7 - 9i|$  is  $k$  then  $k^2 =$
72. The value of  $\tan 78^\circ \tan 42^\circ - \tan 12^\circ \tan 48^\circ =$

73. Consider a polynomial  $p(x)$  of the least degree that has a maximum equal to 6 at  $x=1$  and a minimum equal to 2 at  $x=3$  then the value of  $p(2) + p'(0) - 7$  is
74. A line with direction ratios  $(2,1,2)$  intersects the lines  $\vec{r} = -\vec{j} + \lambda(\vec{i} + \vec{j} + \vec{k})$  and  $\vec{r} = -\vec{i} + \mu(2\vec{i} + \vec{j} + \vec{k})$  at A and B respectively then length of AB is
75. If the shortest distance between the lines  $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$  and  $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$  is  $\lambda\sqrt{30}$  units then the value of  $\lambda$  is

## KEY SHEET

### PHYSICS

|    |     |    |    |    |   |    |    |    |    |
|----|-----|----|----|----|---|----|----|----|----|
| 1  | 4   | 2  | 3  | 3  | 4 | 4  | 3  | 5  | 4  |
| 6  | 1   | 7  | 2  | 8  | 4 | 9  | 2  | 10 | 4  |
| 11 | 4   | 12 | 1  | 13 | 2 | 14 | 1  | 15 | 1  |
| 16 | 1   | 17 | 1  | 18 | 1 | 19 | 4  | 20 | 3  |
| 21 | 223 | 22 | 10 | 23 | 6 | 24 | 10 | 25 | 48 |

### CHEMISTRY

|    |    |    |   |    |    |    |   |    |    |
|----|----|----|---|----|----|----|---|----|----|
| 26 | 2  | 27 | 2 | 28 | 4  | 29 | 3 | 30 | 3  |
| 31 | 3  | 32 | 1 | 33 | 1  | 34 | 2 | 35 | 4  |
| 36 | 2  | 37 | 4 | 38 | 1  | 39 | 3 | 40 | 2  |
| 41 | 2  | 42 | 2 | 43 | 2  | 44 | 4 | 45 | 4  |
| 46 | 16 | 47 | 3 | 48 | 45 | 49 | 6 | 50 | 12 |

### MATHEMATICS

|    |    |    |   |    |   |    |   |    |   |
|----|----|----|---|----|---|----|---|----|---|
| 51 | 4  | 52 | 2 | 53 | 3 | 54 | 4 | 55 | 1 |
| 56 | 2  | 57 | 1 | 58 | 2 | 59 | 1 | 60 | 4 |
| 61 | 4  | 62 | 1 | 63 | 3 | 64 | 4 | 65 | 2 |
| 66 | 1  | 67 | 1 | 68 | 2 | 69 | 2 | 70 | 4 |
| 71 | 18 | 72 | 4 | 73 | 6 | 74 | 3 | 75 | 3 |



## SOLUTIONS PHYSICS

$$1. \quad F = \frac{E}{VT} = [E^1 V^{-1} T^{-1}]$$

$$\alpha = 1, \beta = -1, \gamma = -1$$

$$2. \quad \tau = MB \sin \theta$$

$$\tau \alpha \sin \theta$$

$$\frac{\tau_1}{\tau_2} = \frac{\sin \theta_1}{\sin \theta_2} \Rightarrow \frac{\tau}{\tau/2} = \frac{\sin 90^\circ}{\sin \theta_2}$$

$$\theta_2 = 30^\circ$$

$$\text{Angle of rotation} = 90 - 30 = 60^\circ$$

$$3. \quad \int_{0.1}^{\infty} -\frac{mg}{2x^2} dx = -\frac{1}{2} mv_0^2$$

$$v_0 = 10 \text{ m/s}$$

$$4. \quad a_A = \frac{v_0^2}{R}$$

$$V_A = 2v_0$$

$$R_C = \frac{v_A^2}{a_A} = 4R$$

$$5. \quad T = 2\pi \sqrt{\frac{l}{g}}; T_M = 2\pi \sqrt{\frac{l}{g}}$$

$$\Delta l = \frac{Mgl}{YA}$$

$$l' - l = \frac{Mgl}{YA}$$

$$\frac{1}{Y} = (l' - l) \frac{A}{Mgl}$$

$$= \left[ \frac{T_M^2}{T^2} - 1 \right] \frac{A}{Mg}$$

$$6. \quad W_{AC} = P_0 V_0 \left( \frac{V}{V_0} \right)$$

$$2P_0 V_0 \ln \left[ \frac{P_0}{P} \right] = P_0 V_0 \ln(3)$$

$$P = \frac{P_0}{\sqrt{3}}$$

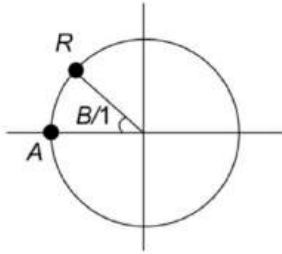
$$7. \quad PV = RT$$

$$P = R \left[ \frac{T_0 + \alpha V^2}{V} \right]$$

$$\frac{dP}{dV} = 0$$

$$\Rightarrow -\frac{T_0}{V^2} + \alpha = 0$$

8.



$$\frac{2\pi}{T}t = \frac{\pi}{12}$$

$$t = \frac{T}{24}$$

9. 
$$\int dE = \int_0^{\pi/2} \frac{\lambda R d\theta \cos \theta}{4\pi\epsilon_0 R^2}$$

$$E = \frac{\lambda}{4\pi\epsilon_0 R} \sin \theta \Big|_0^{\pi/2}$$

$$E = \frac{\sqrt{2}\lambda}{4\pi\epsilon_0 R} = \frac{\lambda}{2\sqrt{2}\pi\epsilon_0 R}$$

10. Since equivalent capacitance increases, charge on capacitor 4 increases. By KVL, charge on capacitor 2 decreases.

11.  $R_{eq} = 2\Omega$

For max power,  $R = 2\Omega$

$$\frac{E_{eq}}{2} = 3 \Rightarrow E_{eq} = 6V$$

$$I = \frac{6}{4} = 1.5A$$

$$P_0 = (1.5)^2 \times 2 = 4.5W$$

12. Inductance can be set to be analogous to mass, as it poses inertia to current change in the electrical circuit.

13. 
$$V_C = \frac{2V_0}{3} \left( 1 - e^{\frac{-3t}{2RC}} \right)$$

$$V_C = \frac{2V_0}{3} \left[ 1 - \frac{1}{3} \right]$$

$$V_C = \frac{4V_0}{9}$$

$$I = \frac{V_C}{2R} = \frac{2V_0}{9R}$$

14.  $V = 5 + 3\sin(\omega t)$

$$V_{rms} = \sqrt{5^2 + \frac{3^2}{2}} = \sqrt{\frac{59}{2}}$$

15.  $\vec{E} \times \vec{B}$  is along positive Z-direction.  $\vec{B}$  is along positive  $\hat{j}$  direction.

$$16. \quad \pi - 2\theta_c = \frac{\pi}{2} \Rightarrow \theta_c = \frac{\pi}{4}$$

$$\frac{1}{\mu} = \sin(\theta_c) \Rightarrow \mu = \sqrt{2}$$

$$17. \quad \beta_{\max} = \frac{\lambda D}{d_0 - a_0}$$

$$\beta_{\min} = \frac{\lambda D}{d_0 + a_0}$$

$$18. \quad \frac{1}{\lambda} = R \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \Rightarrow \frac{1}{\lambda} = R \left[ \frac{1}{9} - \frac{1}{25} \right] \Rightarrow \lambda = \frac{225}{16R}$$

$$19. \quad R_{eq} = 6\Omega$$

$$I = \frac{V_0}{R_{eq}} = 4A$$

$$20. \quad \text{Reading} = 17 + 8 \times 0.1 - 2 \times 0.1 = 17.6 \text{ mm}$$

$$21. \quad \text{L.C.} = 0.01 \text{ mm, zero error} = -0.08 \text{ mm}$$

$$\text{Reading} = 2 \text{ mm} + 15 \times 0.01 + 0.08 = 2.23 \text{ mm}$$

$$22. \quad f = T$$

$$2T = 0.50 \times 4 \times 10 \Rightarrow T = 10N$$

$$23. \quad mv \frac{dv}{dt} = P$$

$$\frac{m}{t} \frac{v^2}{2} \bigg|_{v_0}^{2v_0} = P$$

$$24. \quad I = \int dI = \frac{2\pi}{R} \int_0^R \sigma_0 r^4 dr$$

$$T = \frac{2\pi\sigma_0 R^5}{5R} = \frac{2\pi\sigma_0 R^4}{5}$$

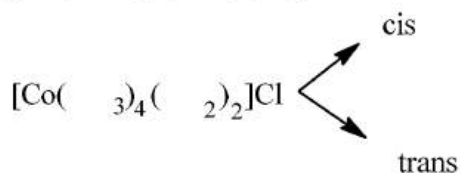
$$25. \quad v_1 = \frac{v_0}{2}$$

$$\text{Radius of curvature at farthest position} = \frac{2(x_0)(2x_0)}{3} = \frac{4x_0}{3}$$

$$a_{\min} = \frac{\left[ \frac{v_0}{2} \right]^2}{\frac{4x_0}{3}} = \frac{3v_0^2}{16x_0}$$

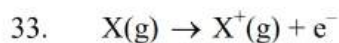
## CHEMISTRY

26. 1.6 gm oxide loses 0.16 gm  
 80 gm oxide loses = 8 gm 'O' =  $\frac{1}{2}$  mole of 'O'  
 $\text{TiO}_{\left(2-\frac{1}{2}\right)} = \text{TiO}_3 = \text{Ti}_2\text{O}_3$
27.  $\text{Zn} + \text{Ni}^{+2} \longrightarrow \text{Zn}^{+2} + \text{Ni}$   
 $E^\circ = E^\circ_{\text{Ni}^{+2}/\text{Ni}} - E^\circ_{\text{Zn}^{+2}/\text{Zn}}$   
 $= -0.23 - (-0.76) = +0.53 \text{ V}$   
 Positive value shows that the process is spontaneous.  
 Rest of all (I) (II) (III) combination have negative  $E^\circ$  value.  
 (I)  $E^\circ = -0.44 - (-0.23) = -0.21 \text{ V}$   
 (II)  $E^\circ = -0.76 - (-0.23) = -0.53 \text{ V}$   
 (III)  $E^\circ = -0.76 - (-0.44) = -0.32 \text{ V}$
28. HINT: Assume rate law  
 $r = K[\text{H}_3\text{AsO}_4]^x [\text{H}_3\text{O}^+]^y [\text{I}^-]^z$   
 Solving by the help of various experiments  
 $x = 1, \quad y = 2 \quad \text{and} \quad z = 1$   
 total order = 4
29.  $\Delta T_f = i K_f m$   
 $3.72 = i \times 1.86 \times 0.4$   
 $i = 5 \therefore n = 4$
30. Hint: Due to extra stability of half-filled f-subshell.
31.  $[\text{Co}(\text{NH}_3)_4(\text{ONO})_2]\text{Cl}$  = linkage isomers  
 $[\text{Co}(\text{NH}_3)_4(\text{NO}_2)(\text{Cl})]\text{NO}_2$  = ionization isomers



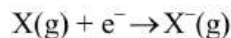


$$\Delta_c H = \left[ \begin{array}{l} 8 \times \text{B.E. (C-H)} \\ + 2 \times \text{B.E. (C-C)} \\ + 5 \times \text{B.E. (O=O)} \end{array} \right] - \left[ \begin{array}{l} 6 \times \text{B.E. (C=O)} \\ + 8 \times \text{B.E. (O-H)} \\ + 3 \times |\text{R.E.}| \text{ of } CO_2 \\ + 4 \times \Delta_{\text{vap}} H(H_2O) \end{array} \right]$$



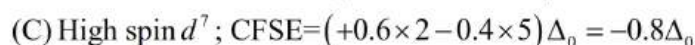
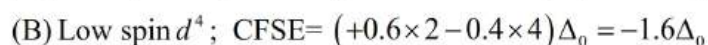
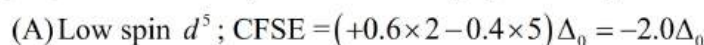
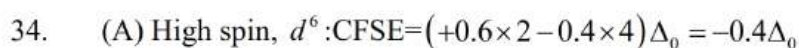
If I.E. is ionisation enthalpy, then

$$\therefore \frac{N_0}{2} (\text{I.E.}) = E_1 \quad \therefore \text{I.E.} = \frac{2E_1}{N_0}$$



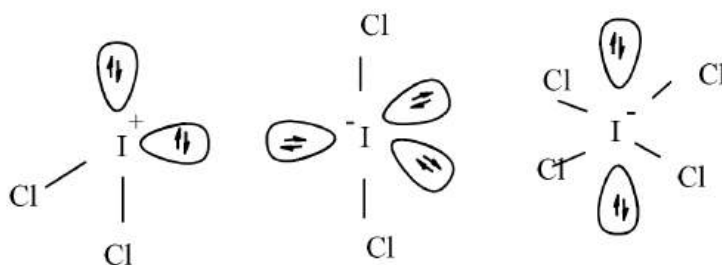
If  $\Delta_{\text{eg}} H$  is electron gain enthalpy, then

$$\therefore 2N_0(\text{E.A.}) = -E_2 \quad \therefore \Delta_{\text{eg}} H = -\frac{E_2}{2N_0}$$



Magnitude of CFSE is maximum in (B)

35.

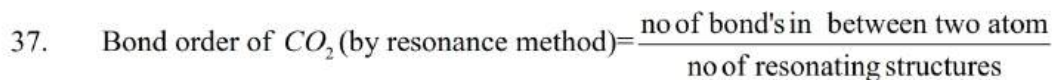


$$\text{Unused acid required} = 40 \text{ mL } \frac{N}{10} \text{ NaOH}$$

$$40 \text{ mL } \frac{N}{10} \text{ NaOH} = 40 \text{ mL } \frac{N}{10} H_2SO_4 = 20 \text{ mL } \frac{N}{5} H_2SO_4$$

$$\text{Acid used for absorption of ammonia} = (60-20) \text{ mL } \frac{N}{5} H_2SO_4 = 40 \text{ mL } \frac{N}{5} H_2SO_4$$

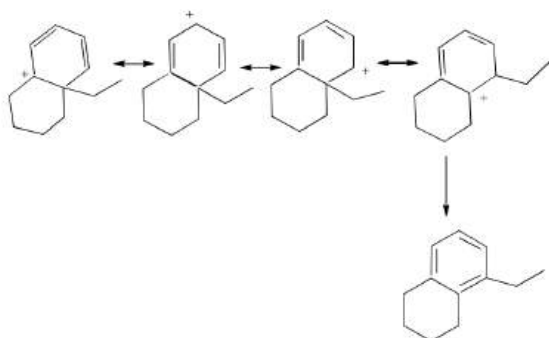
$$\text{Percentage of nitrogen} = \frac{1.4 \times N_1 \times V}{W} = \frac{1.4 \times \frac{1}{5} \times 40}{0.2} = \frac{1.4 \times 40}{0.2 \times 5} = 56\%$$



$$\text{i.e., bond order in } CO_3^{2-} \text{ (by resonance method)} = \frac{4}{3} = 1.33$$

$$\text{Bond length} \propto \frac{1}{\text{Bond order}}$$

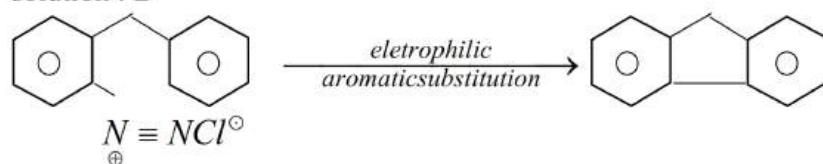
38.



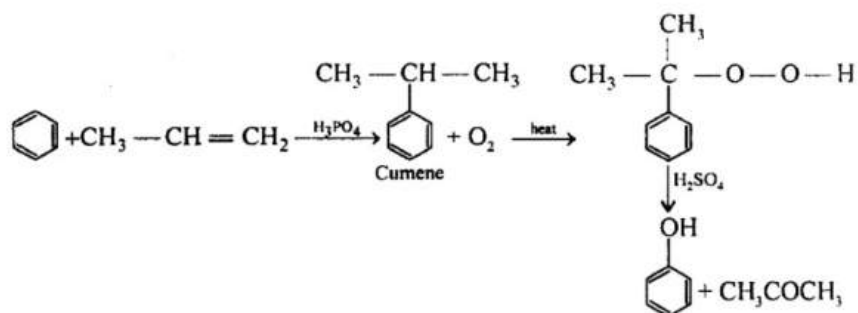
39. The compound A, despite a tertiary alcohol, cannot be readily converted into chloride because OH is present at bridge head. The compound C, allyl alcohol can be readily converted into allyl chloride, whose formation is responsible for white cloudiness.
40.  $t-BuO^- \rightarrow$  astearic ally hindered base will give Hofmann elimination as major product. Where as  $EtO^-$  will give Saytzeffs product.

41.

solution : B



42.

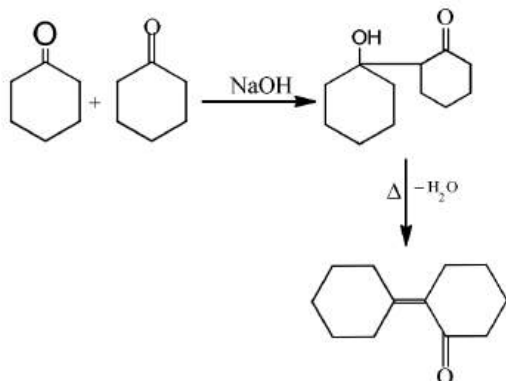


(A) is not possible because  $CH_3-CH_2-\overset{+}{C}H_2$  is less stable than  $CH_3-\overset{+}{C}H-CH_3$

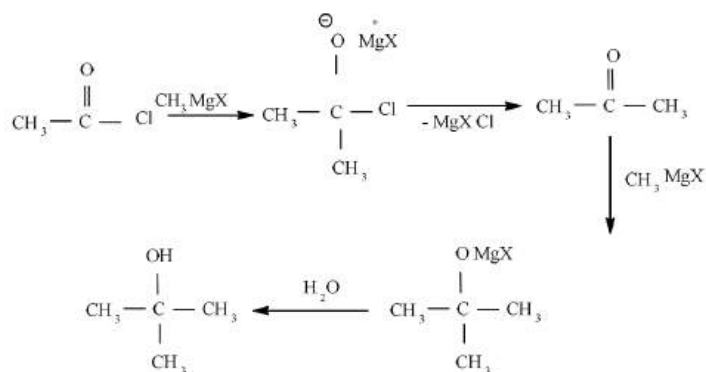
(C) is not possible because acetophenone and  $CH_3OH$  cannot be formed.

(D) is not possible because  $CH_3-\overset{OH}{\underset{|}{CH}}-CH_3$

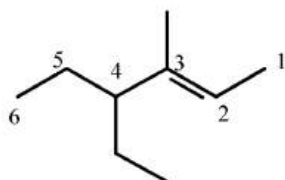
43.



44.



45.

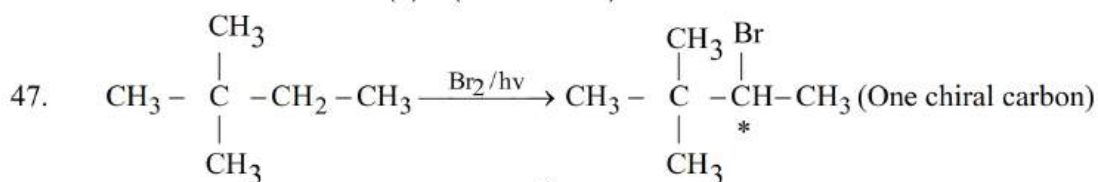


4-Ethyl -3-methylhex-2-ene

46. Velocity of an electron in  $\text{He}^+$  ion in an orbit =  $\frac{2\pi Ze^2}{nh}$  .....(i)Radius of  $\text{He}^+$  ion in an orbit =  $\frac{n^2 h^2}{4\pi^2 m e^2 Z}$  ....(ii)

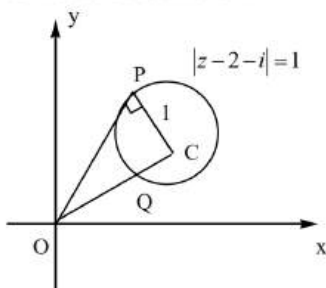
By equations (i) and (ii),

$$\begin{aligned}
 \text{Angular velocity } (\omega) &= \frac{u}{r} = \frac{8\pi^3 Z^2 m e^4}{n^3 h^3} \quad \text{....(iii)} \\
 &= \frac{8 \times (22/7)^3 \times (2)^2 \times (9.108 \times 10^{-28}) \times (4.803 \times 10^{-10})^4}{(2)^3 \times (6.626 \times 10^{-26})^3} = 2.067 \times 10^{16} \text{ sec}^{-1}.
 \end{aligned}$$

48. Number of possible dipeptides is  $(3)^2 = 9$   
 $5 \times 9 = 45$ 49. Basicity of  $\text{H}_3\text{PO}_4$ ,  $\text{H}_3\text{PO}_3$  and  $\text{H}_3\text{PO}_2$  are 3, 2, 1 respectively. $\therefore$  sum of basicity = 650. On dilution (addition of water) pH of the buffer solution will not change therefore  $x=0$  and  $x+12=12$

## MATHS

51. Consider statement-2



Q is the point on the circle which is nearest to the origin

$$OQ = OC - CQ = \sqrt{5} - 1$$

Statement-2 is true

Consider statement-1

Point on the circle with maximum argument is 1

$$\text{If } \angle COP = \alpha \Rightarrow \sin \alpha = \frac{1}{\sqrt{5}} \Rightarrow \tan \alpha = \frac{1}{2}$$

$$\text{and } \arg(z_0) = 2\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

Statement-1 is false.

53. Let  $\alpha_1, \alpha_2$  and  $\alpha_3$  be the roots of  $f(x) = 0$

Such that  $\alpha_1 < \alpha_2 < \alpha_3$  and  $g(x)$  takes all values from  $[-6, \infty)$

$$g(x) = (x+1)^2 - 6 \geq -6$$

$$\alpha_3 \leq -7, \alpha_2 \leq -8, \alpha_1 \leq -9$$

$$\therefore a + b + c \geq 719$$

Minimum value of  $a + b + c$  is 719

$$\therefore \alpha_1 + \alpha_2 + \alpha_3 = -a \Rightarrow -a \leq -24 \Rightarrow a \geq 24$$

$$\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1 = b \Rightarrow b = 191$$

$$\alpha_1\alpha_2\alpha_3 = -c$$

$$-c = -504$$

$$a + b + c \geq 719$$

54. Make 1 group of 2 persons, 1 group of 4 persons and 3 groups of 3 persons among 15 persons

(except 2 particular persons) hence the number of ways by grouping method is  $\frac{15!}{2!4!(3!)^3 3!}$

Now we add that 2 persons in the group of 2 persons and thus the number of arrangements of these groups into cars and autos is  $\frac{15!}{2!4!(3!)^3 3!} \times 2! \times 3! = \frac{15!}{4!(3!)^3}$

55.  $E_1$  be the event that the answers is guessed  
 $E_2$  be the event that the answer is copied  
 $E_3$  be the event that the examinee knows the answer and  
 $E$  be the event that the examinee answer correctly

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{6}$$

Assume that event  $E_1, E_2$  and  $E_3$  are exhaustive  $P(E_1) + P(E_2) + P(E_3) = 1$

$$\Rightarrow P(E_3) = \frac{1}{2}$$

$$P\left(\frac{E}{E_1}\right) = \frac{1}{4}, P\left(\frac{E}{E_2}\right) = \frac{1}{8}, P\left(\frac{E}{E_3}\right) = 1 \text{ (Probability of answering correctly by knowing)}$$

$$\therefore P\left(\frac{E_3}{E}\right) = \frac{P(E_3)P\left(\frac{E}{E_3}\right)}{P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + P(E_3)P\left(\frac{E}{E_3}\right)} = \frac{24}{29}$$

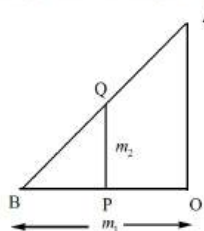
56.  $P(E) = P(2) + P(3) + P(5) + P(7) = 0.62$

$$P(F) = P(1) + P(2) + P(3) = 0.50$$

$$P(E \cap F) = P(2) + P(3) = 0.35$$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) \\ = 0.62 + 0.50 - 0.35 = 0.77$$

58. Let  $BP = x$  from the similar triangle properly



$$\frac{AO}{m_1} = \frac{m_2}{x} \Rightarrow AO = \frac{m_1 m_2}{x}$$

$$\frac{d(AO)}{dt} = \frac{-m_1 m_2}{x^2} \frac{dx}{dt}$$

$$\text{When } x = \frac{m_1}{2}$$

$$\frac{d(AO)}{dt} = -\frac{2m_2}{5} m/s$$

62. Let  $I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})} \rightarrow 1$

By kings rule

$$I = \int_0^1 \frac{e^{2-4x} dx}{(5+2x-2x^2)(1+e^{2-4x})} \rightarrow 2$$

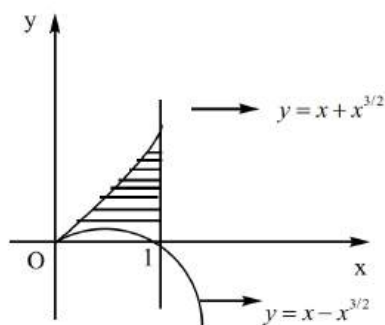
Adding 1 and 2

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{5-2(x^2-x)} = \int_0^1 \frac{dx}{\frac{11}{4} - \left(x - \frac{1}{2}\right)^2} = \frac{1}{2\sqrt{11}} \left| \ln \frac{\frac{\sqrt{11}}{2} + x - \frac{1}{2}}{\frac{\sqrt{11}}{2} - \left(x - \frac{1}{2}\right)} \right|_0^1$$

$$\frac{1}{2\sqrt{11}} \left| 2 \ln \left( \frac{\sqrt{11}+1}{\sqrt{11}-1} \right) \right| = \frac{1}{\sqrt{11}} \ln \left( \frac{\sqrt{11}+1}{\sqrt{11}-1} \right)$$

$$\Rightarrow I = \frac{1}{2\sqrt{11}} \ln \frac{(\sqrt{11}+1)^2}{10}$$

63.



$$(y-x)^2 = x^3 \Rightarrow y-x = \pm x^{3/2}$$

$$\Rightarrow y = x \pm x^{3/2}$$

$$y = x + x^{3/2} \rightarrow 1, y = x - x^{3/2} \rightarrow 2$$

1 is an increasing function

2 meets x=axis at x=0,1

$$\text{Required area} = \int_0^1 [(x+x^{3/2}) - (x-x^{3/2})] dx$$

$$= 2 \int_0^1 x^{3/2} dx = 2 \left[ \frac{2}{5} x^{5/2} \right]_0^1 = \frac{4}{5} \text{ sq. units}$$

64.  $\{x\} = x - [x]$  which is periodic with period 1.

Statement 2 is true.

Consider Statement 1.

$$f(x) = \sin(3x - [3x]) = \sin(\{3x\})$$

Using Statement 2, period of  $f(x)$  is  $\frac{1}{3}$ .

Statement 1 is false.

65. Let 1<sup>st</sup> term be 'a<sub>1</sub>' and common ratio be 'r' then  $a_1 + a_1r + a_1r^2 + \dots + a_1r^{100} = 125$

$$\Rightarrow \frac{a_1(1-r^{101})}{1-r} = 125 \quad (\text{let } 0 < r < 1)$$

$$\sum_{r=1}^{101} \left( \frac{1}{a_i} \right) = \frac{1}{a_1} + \frac{1}{a_1r} + \frac{1}{a_1r^2} + \dots + \frac{1}{a_1r^{100}} = \frac{\frac{1}{a_1} \left( \left( \frac{1}{r} \right)^{101} - 1 \right)}{\frac{1}{r} - 1} \quad \left( \text{here } \frac{1}{r} > 1 \right)$$

$$= \frac{(1-r^{101})}{a_1r^{100}(1-r)} = \frac{1}{a_1r^{100}} \times \frac{125}{a_1} = \frac{125}{(a_1r^{50})^2} = \frac{125}{(a_{51})^2} = \frac{125}{(25)^2} = \frac{1}{5}$$

66. Last term expansion is  ${}^nC_n \left( -\frac{1}{\sqrt{2}} \right)^n = \left( \frac{1}{3\sqrt[3]{9}} \right)^{\log_3 8}$

$$\Rightarrow (-1)^n \left( \frac{1}{2} \right)^{n/2} = \left( \frac{1}{3^{5/3}} \right)^{\log_3 8} = 3^{-\frac{5}{3} \times 3 \times \log_3 2}$$

$$= 3^{-5 \log_3 2} = 3^{\log_3 2^{-5}} = 2^{-5} = \left( \frac{1}{2} \right)^5 \Rightarrow n = 10$$

$$\therefore 5^{\text{th}} \text{ term from the beginning } {}^{10}C_4 \left( \sqrt[3]{2} \right)^6 \left( -\frac{1}{\sqrt{2}} \right)^4 = {}^{10}C_4 2^2 \cdot \frac{1}{2^2} = {}^{10}C_4$$

67. Let the two numbers be a, b

$$\therefore \bar{x} = \frac{2+4+10+12+14+a+b}{7} = 8$$

$$\Rightarrow a+b=14 \rightarrow 1$$

$$\sigma^2 = \frac{\sum x_i^2}{N} - \left( \frac{\sum x_i}{n} \right)^2 = 16$$

$$\Rightarrow 460 + a^2 + b^2 = (16+64) \times 7 \Rightarrow a^2 + b^2 = 100 \rightarrow 2$$

From 1 and 2,  $a-b=2 \rightarrow 3$

$$a=8, b=6$$

68.  $a \{ \sin(\theta + \phi) - \sin(\theta - \phi) \} = b \{ \sin(\theta - \phi) + \sin(\theta + \phi) \}$

$$\Rightarrow 2a \sin \phi \cos \theta = 2a \sin \theta \cos \phi$$

$$\Rightarrow a \tan \phi = b \tan \theta$$

$$\frac{2a \tan \frac{\phi}{2}}{1 - \tan^2 \frac{\phi}{2}} = \frac{2b \tan \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} \rightarrow 1$$

$$a \tan \frac{\theta}{2} - b \tan \frac{\phi}{2} = c \Rightarrow \tan \frac{\theta}{2} = \frac{b \tan \frac{\phi}{2} + c}{a} \rightarrow 2$$

$$\text{From 1 and 2 we get } \tan \frac{\phi}{2} (a^2 - b^2 - c^2) = bc \left( 1 + \tan^2 \frac{\phi}{2} \right)$$

$$\Rightarrow \sin \phi = \frac{2 \tan \frac{\phi}{2}}{1 + \tan^2 \left( \frac{\phi}{2} \right)} = \frac{2bc}{a^2 - b^2 - c^2}$$

69. Let  $\sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + \sqrt{(\tan x - \sin x) + \dots}}}$   
 $\Rightarrow y = \sqrt{(\tan x - \sin x) + y}$   
 $\Rightarrow y^2 - y - (\tan x - \sin x) = 0$   
 $\Rightarrow y = \frac{1 + \sqrt{1 + 4(\tan x - \sin x)}}{2}$   
 Again  $z = \sqrt{x^3 + \sqrt{x^3 + \sqrt{x^3 + \dots}}}$   
 $= \sqrt{x^3 + z} \Rightarrow z^2 - z - x^3 = 0$   
 $\Rightarrow z = \frac{1 + \sqrt{1 + 4x^3}}{2}$   
 $\lim_{x \rightarrow 0^+} \frac{-1 + \sqrt{1 + 4(\tan x - \sin x)}}{-1 + \sqrt{1 + 4x^3}} = \lim_{x \rightarrow 0^+} \frac{4(\tan x - \sin x) + (1 + \sqrt{1 + 4x^3})}{4x^3(1 + \sqrt{1 + 4(\tan x - \sin x)})}$   
 $= \lim_{x \rightarrow 0^+} \frac{\left(\frac{\sin x}{\cos x} - \frac{\sin x}{1}\right)(1 + \sqrt{1 + 4x^3})}{x^3(1 + \sqrt{1 + 4(\tan x - \sin x)})} = \lim_{x \rightarrow 0^+} \frac{\sin x(1 - \cos x)}{x^3 \cos x} \times \frac{1 + \sqrt{1 + 4x^3}}{1 + \sqrt{1 + 4(\tan x - \sin x)}}$   
 $= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \frac{2 \sin^2 x / 2}{4x^2 / 4} \cdot \frac{1}{\cos x} \cdot \frac{1 + \sqrt{1 + 4x^3}}{1 + \sqrt{1 + 4(\tan x - \sin x)}} = 1 \cdot \frac{1}{2} \cdot 1 \cdot \frac{1 + 1}{1 + 1} = \frac{1}{2}$

70. Put  $x = 2 \cos \theta, y = 3 \sin \theta$   
 $\sin^{-1}(1) + \cos^{-1}\left(\frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta - 2\right)$   
 $\sin^{-1}(1) + \cos^{-1}\left(\cos\left(\theta - \frac{\pi}{4}\right) - 2\right)$   
 $-1 \leq \cos\left(\theta - \frac{\pi}{4}\right) - 2 \leq 1$   
 $1 \leq \cos\left(\theta - \frac{\pi}{4}\right) \leq 1 \Rightarrow \cos\left(\theta - \frac{\pi}{4}\right) = 1$   
 $= \frac{\pi}{2} + \cos^{-1}(1 - 2) = \frac{\pi}{2} + \cos^{-1}(-1) \Rightarrow \frac{\pi}{2} + \pi = \frac{3\pi}{2}$

71. Let  $z = x + iy$   
 $z - z_1 = (x - 10) + (y - 6)i$   
 $z - z_2 = (x - 4) + (y - 6)i$   
 $\arg\left(\frac{z - z_1}{z - z_2}\right) = \frac{\pi}{4}$   
 $\Rightarrow \tan^{-1}\left[\frac{6(y - 6)}{(x - 10)(x - 4) + (y - 6)^2}\right] = \frac{\pi}{4}$   
 $\Rightarrow x^2 + y^2 - 14x - 18y + 112 = 0$   
 $|z - 7 - 9i|^2 = (x - 7)^2 + (y - 9)^2 \Rightarrow x^2 - 14x + y^2 - 18y + 130 = -112 + 130 = 18$

73. The polynomial is every where differentiable function  
 $\therefore$  The points of extremum can only be the no. of derivative  
 The derivative of polynomial is  $P'(x) = a(x-1)(x-3) = a(x^2 - 4x + 3)$   
 $P(1) = 6$

$$P(x) = \int_1^x P'(x) dx + 6 = a \left( \frac{x^3}{3} - 2x^2 + 3x - \frac{4}{3} \right) + 6$$

$$\text{Also } P(3) = 6 \Rightarrow a = 3$$

$$P(x) = x^3 - 6x^2 + 9x + 2$$

$$\therefore P(2) = 4, P'(0) = 9$$

$$\therefore P(2) + P'(0) - 7 = 6$$

74.  $L_1 = \frac{x-0}{1} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda$ ,  $L_2 = \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \mu$

Any point on  $L_1$  and  $L_2$  be  $(\lambda, \lambda-1, \lambda)$  and  $(2\mu-1, \mu, \mu)$  respectively.

$$\frac{2\mu-1-\lambda}{2} = \frac{\mu-\lambda+1}{1} = \frac{\mu-\lambda}{2}$$

On solving  $\mu = 1, \lambda = 3$

$$A = (3, 2, 3), B = (1, 1, 1)$$

$$AB = \sqrt{4+1+4} = 3$$

75. Shortest distance =  $\frac{\left| \begin{bmatrix} \bar{a}-\bar{c} & \bar{b} & \bar{d} \end{bmatrix} \right|}{\left| \bar{b} \times \bar{d} \right|}$

$$\text{Here } \bar{a} = 3\bar{i} + 8\bar{j} + 3\bar{k}, \bar{b} = 3\bar{i} - \bar{j} + \bar{k}, \bar{c} = -3\bar{i} - 7\bar{j} + 6\bar{k}, \bar{d} = -3\bar{i} + 2\bar{j} + 4\bar{k}$$

$$\therefore \text{Shortest distance} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30} = \lambda\sqrt{30} \Rightarrow \lambda = 3$$